

OLIMPIADA DE MATEMATICĂ A SATELOR DIN ROMÂNIA

ETAPA JUDEȚEANĂ

Barem CLASA a VI -a

Problema 1.(7 puncte)

a)

$$a_{1999} + a_{2000} = 1999 + 1999^2 + 2000 + 2000^2 \quad (1p)$$

$$= 1999(1999 + 1) + 2000(2000 + 1)$$

$$= 2000(1999 + 2001) \quad (1p)$$

$$= 2000 \cdot 4000 = 8000000 = 200^3.$$

b) $P = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 = 2 \cdot (1 + 2) \cdot 4 \cdot (1 + 4) \cdot 6 \cdot (1 + 6) \cdot 8 \cdot (1 + 8) \cdot 10 \cdot (1 + 10) \quad (2p)$

$$P = (2 + 2^2) \cdot (4 + 4^2) \cdot (6 + 6^2) \cdot (8 + 8^2) \cdot (10 + 10^2) \quad (1p)$$

$$P = a_2 \cdot a_4 \cdot a_6 \cdot a_8 \cdot a_{10} \quad (1p)$$

Problema 2.(7 puncte)

$$a:b=c \text{ rest } 4$$

$$c+r=30$$

$$r=3c+2 \quad (1p)$$

$$c+3c+2=30$$

$$4c+2=30$$

$$4c=28$$

$$c=28:4=7$$

$$r=3c+2 \Rightarrow 3 \times 7 + 2 = 23 \quad (2p)$$

$$a+b=1023 \Rightarrow 7b+23+b=1023$$

$$8b=1000$$

$$b=1000:8$$

$$b=125 \quad (1p)$$

$$a=bx+c+r$$

$$b=125 \quad c=7 \quad r=23 \quad (2p)$$

$$a=7b+r=7b+23$$

$$a=7 \times 125 + 23$$

$$a=898 \quad (1p)$$

Problema 3.(7 puncte)

a) $11a + 33b = 25a + 5b \Rightarrow 2b = a \Rightarrow b = 50\% \text{ din } a \dots\dots\dots(2p)$

b) $\frac{b}{a} = \frac{1}{2} \dots\dots\dots(1p)$

$$10b + 5c = 7b + 14c \Rightarrow b = 3c \Rightarrow \frac{c}{b} = \frac{1}{3} \dots\dots\dots(1p)$$

c) $b = 9; c = 3; a = 18 \dots\dots\dots(3p)$

Problema 4.(7 puncte)

$$x+(x+30^0)+(x+30^0+30^0)+(x+30^0+30^0+30^0)+\dots+(x+(n-1)\cdot 30^0)=360^0 \text{ (1p)}$$

$$n \cdot x + (1 + 2 + 3 + 4 + \dots + (n-1)) \cdot 30^0 = 360^0$$

$$n \cdot x + \frac{(1+(n-1)) \cdot (n-1)}{2} \cdot 30^0 = 360^0 \text{ (2p)}$$

$$\Rightarrow 2n \cdot x + n \cdot (n-1)30^0 = 720^0 : 2 \Rightarrow$$

$$n \cdot x + n \cdot (n-1) \cdot 15^0 = 360^0, n \neq \{0,1,2\} \text{ (număr natural) (1p)}$$

$$\text{Considerăm } n=3 \Rightarrow 3(x+2 \cdot 15^0) = 360^0 : 3 \Rightarrow x = 90^0$$

$$\text{Fie } n=4 \Rightarrow 4(x+3 \cdot 15^0) = 360^0 : 4 \Rightarrow x = 45^0 \text{ (1p)}$$

$$n=5 \Rightarrow 5(x+4 \cdot 15^0) = 360^0 : 5 \Rightarrow x = 12^0$$

$$n=6 \Rightarrow 6(x+5 \cdot 15^0) = 360^0 : 6 \Rightarrow x+5 \cdot 15^0 = 60^0 \text{ contradicție (1p)}$$

$\Rightarrow$ numărul maxim de unghiuri care se formează în jurul punctului este $n=5$,(1p)

iar unghiurile sunt $\{12^0, 42^0, 72^0, 102^0, 132^0\}$